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LOW-COST VAULTED EARTHEN MASONRY FLOOR SYSTEMS

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Abstract

Rapid global urbanization is expected to add 2.5 billion urban dwellers by 2050, particularly in Less Economically Developed Countries (LEDCs), presenting a challenge in meeting affordable housing demands. The prevalent use of inefficient reinforced concrete frames in LEDCs, relying on imported cement and steel, increases construction costs and carbon emissions. This paper proposes earthen vaulted floor systems as an alternative, analyzing three typologies for spanning capacity. Furthermore, a case study compares the costs and carbon footprints of two typologies with a concrete flat slab. Lastly, a 3m span physical prototype which validates one typology's viability is discussed. Results highlight the structural viability, cost-effectiveness, and low carbon footprint of earthen vaulted floor systems, showcasing up to a 74% (19 kg_{CO2e}/m^2 versus 72 kg_{CO2e}/m^2) reduction in embodied carbon and up to a 62% (\$58/m^2 versus \$153/m^2) reduction in cost compared to typical concrete flat slabs.

Keywords: shallow vaults, vaulted floors, earthen shells, natural materials, affordable housing, sustainable design

1. Introduction

1.1 Motivation

The United Nations projects an additional 2.5 billion people living in cities by 2050 [1]. By 2100, the 20 most populous cities are expected to be in less economically developed countries (LEDCs), with half of the urban growth and 13 of the top 20 cities in Africa alone [2]. The rapid urbanization in LEDCs demands a significant increase in floor area, with the International Energy Agency (IEA) estimating a global built floor area increase of 235 billion m^2 by 2050, with about 80% of the growth through 2030 occurring in emerging economies [3]. These projections underscore the urgent need for affordable housing over the next several decades.

Unlike construction in more economically developed countries, where labor costs take precedence, construction costs in LEDCs are predominantly influenced by material expenses, notably the importation of cement and steel. Studies show that materials can account for 60-90% of the overall cost of new construction in LEDCs [4]-[5].

Globally, the construction of multi-story housing primarily uses reinforced concrete frames with flat slabs, forming the basis for comparison in this paper. Research indicates that for a theoretical 10-story concrete building, most of the structural mass is in the slabs [6]. Similarly, data-driven modeling of over 100,000 parametric-designed concrete buildings reveals that in residential buildings, 48% of a building's embodied carbon (a proxy for material quantity) is attributed to the slabs [7].

Given the material and cost concentration in floor systems, this paper focuses on reducing housing costs in LEDCs by proposing an alternative to the traditional flat slab floor system. Specifically, it introduces an unreinforced barrel-vaulted compression-only masonry floor system. Exploiting the compression-only nature of vaults, this system significantly reduces steel usage while incorporating low-strength materials like earthen masonry units instead of concrete or fired clay bricks, thereby reducing costs compared to traditional floor systems. While this study emphasizes on cost reduction, the decreased reliance on cement and steel also aligns with global goals to reduce CO2 emissions.

1.2 Efficient Floor Systems

The vault's structural advantage lies in its reliance on axial compression rather than bending, allowing for the use of brittle materials like stone and brick to span large distances. Arches and vaults derive strength from their geometry, resulting in low internal stresses compared to the masonry strength, making crushing failure rare [8]. This characteristic led ancient civilizations like the Nubians and Romans to efficiently utilize arches and vaults for spanning structures.

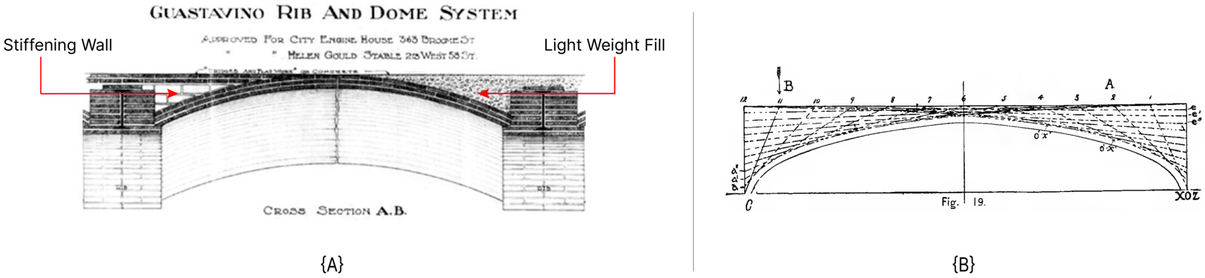

Figure 1: (a) Guastavino Tile Arch Floor System [9]; (b) Thrust line under moving point load is always contained inside the stiffening wall or fill [10]

In the late 19th to mid-20th century, Spanish master builders Rafael Guastavino Sr. and Jr. employed thin clay tiles to create cost-effective vaulted shell roofs and floor systems (Figure 1-A). These shallow tile vaults featured a series of stiffening ribs, on top of which a flat walking surface spanned, or in lieu of the stiffening ribs a lightweight fill was sometimes used. The Guastavino Company constructed thousands of such vaults until the mid-20th century, when they closed due to rising labor costs and the growing popularity of concrete slabs [9].

The challenge of climate change has sparked a renewed interest in leveraging geometry for low-carbon construction techniques. The HiLo floor system (Figure 2-A) by the Block Research Group (BRG) exemplifies this shift, utilizing digital fabrication to create a thin, doubly curved funicular shell floor system with vertical stiffeners, achieving a 70% reduction in concrete and a 90% reduction in steel compared to a conventional slab [11]. Another contemporary example (Figure 2-B) focuses on optimizing the shape of one-way ribbed slabs to minimize material usage, showcasing up to a 50% reduction in embodied energy compared to a flat two-way slab [12].

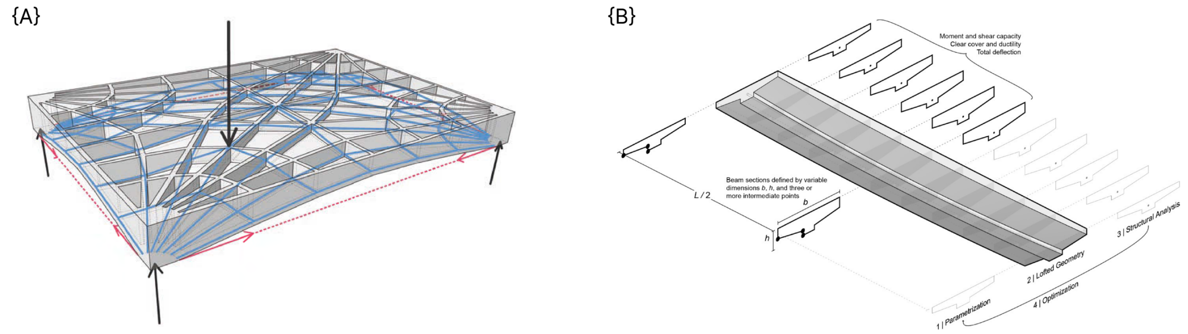

Figure 2: (a) BRG HiLo floor system [11]; (b) Shaped ribbed slab floor system [12]

1.3 Compressed Earth Bricks

Compressed stabilized earth bricks (CSEBs) are by compressing soil, sand, and small amounts of stabilizing agents, such as cement or lime, under pressure. The properties of CSEBs, including density and strength (ranging from 1500-2200 kg/m3 and 1-40 MPa), vary based on factors like soil composition, compaction pressure, and stabilization amount [13].

Recently, there has been growing interest in CSEBs for affordable housing due to their sustainability and cost-effectiveness. These bricks utilize locally available materials, demand lower energy inputs than conventional alternatives, and exhibit reduced embodied carbon. CSEBs typically contain 5-7% cement, in contrast to the 10-15% found in conventional concrete used for structural slabs [13]. Furthermore, unlike fired clay bricks, CSEBs do not require firing, significantly reducing production energy.

A case study by the MASS Design Group on the Rwanda Institute for Conservation Agriculture project compared CSEBs and fired clay bricks, revealing that a 5% CSEB had a global warming potential nearly five times lower than a locally sourced fired clay brick (0.04 kg_{CO2e}/kg vs. 0.20 kg_{CO2e}/kg) [14]. Although the regional cost of CSEBs varies, recent literature consistently demonstrates savings ranging from 15-40% compared to fired clay bricks across various countries [15].

1.4 Problem Statement

Recent construction projects demonstrate significant carbon savings and present attractive choices for environmentally conscious buildings [11], [12], [16]. However, technologies like 3D printing, robotics, or custom high-strength concrete mixtures, which contribute to carbon effectiveness, often come with a cost premium and are not currently economically feasible for many LEDCs [11], [12] [16]. Given the urgent demand for housing in emerging economies, there is a distinct need for flooring systems that reduce carbon emissions and prioritize affordability.

This paper introduces an alternative floor system that combines a vaulted form's shell-like structural behavior and material efficiency with the affordability and sustainability of CSEBs. It aims to address the following questions:

1. How does the spanning capacity of earthen vaulted floor systems perform under typical loads?
2. What are the cost and carbon savings of an earthen vaulted floor versus a concrete flat slab?
3. Are earthen vaulted floor systems structurally viable in full-scale testing?

2. Methodology

These questions are addressed through three subsequent subsections as outlined below.

Floor Typologies & Span Limits: This section explores the viability of three earthen vaulted floor system typologies, employing parametric models. A 2D equilibrium analysis evaluates working stresses under prescribed building code loading for varying spans and aspect ratios.

Case Study: Subsequently, the paper compares two proposed floor system typologies with a slab from a recent building in Ethiopia. Construction costs are calculated using Ethiopian market rates, and a cradle-to-gate (A1-A3) embodied carbon assessment is conducted.

Physical Prototype: The final section assesses the proposed floor system's structural viability, showcasing 2D & 3D equilibrium analysis and experimental results from a recently built 3m span prototype of one of the proposed floor typologies.

3. Span Limits

3.1 Floor Typologies

Figure 3 illustrates three floor typologies featuring a primary 5 cm thick unreinforced CSEB parabolic barrel vault. Typologies SR (sparse ribs) and DR (dense ribs) incorporate stiffening walls, which serve the dual purpose of supporting asymmetrical loads and providing a level surface for the finishing surface to span. Typology SR has 10 cm thick stiffening walls spaced at 1 m, while typology DR has 5 cm walls spaced at 30 cm. In contrast, typology LF (light fill) replaces stiffening walls with a lightweight filler slab.

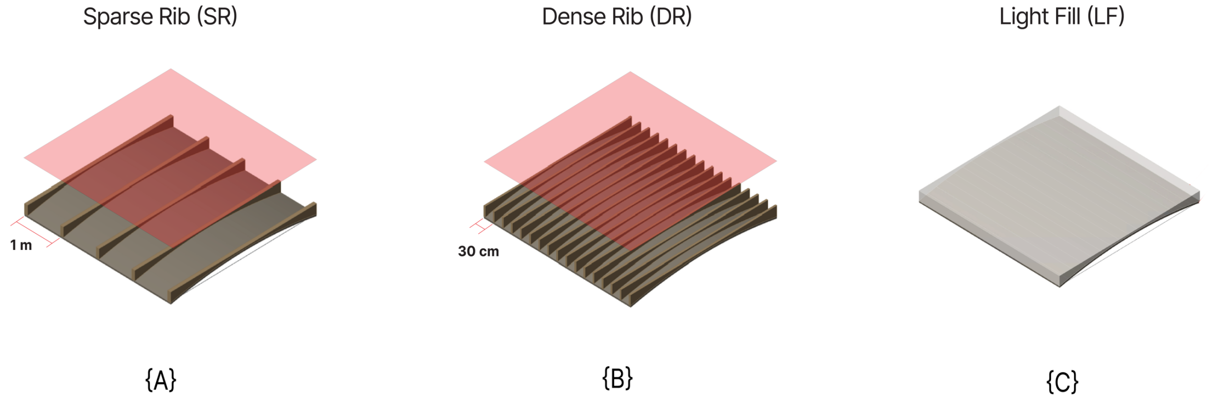

Figure 3: Earthen vaulted floor typologies; red surfaces in (a) and (b) represent imposed finishing surface loads.

All typologies assume a CSEB density of 2000 kg/m^3 based on experimental results and a literature review on CSEBs [13]. In typologies SR and DR, instead of specifying a finishing surface, the analysis imposes finishing surface dead loads of 1 kN/m^2 and 0.5 kN/m^2, respectively. This acknowledges the unique context of each location, where finishing surfaces can vary. For typology LF (Light Fill), no specific material is prescribed for the lightweight fill; instead, the fill density is estimated at 1400 kg/m^3, deemed reasonable for materials like aerated concrete or crushed construction waste. For structural safety, each barrel vault must resolve its outward thrust. While many possible solutions exist for restraining the vault thrust, this paper employs steel ties at each stiffening wall (Figure 5-A), or every 1 m for the LF typology. Alternatives to steel ties include a ring beam or thick buttressing external walls; furthermore, a combination of multiple systems can be employed to increase redundancy.

3.2 Analysis Assumptions

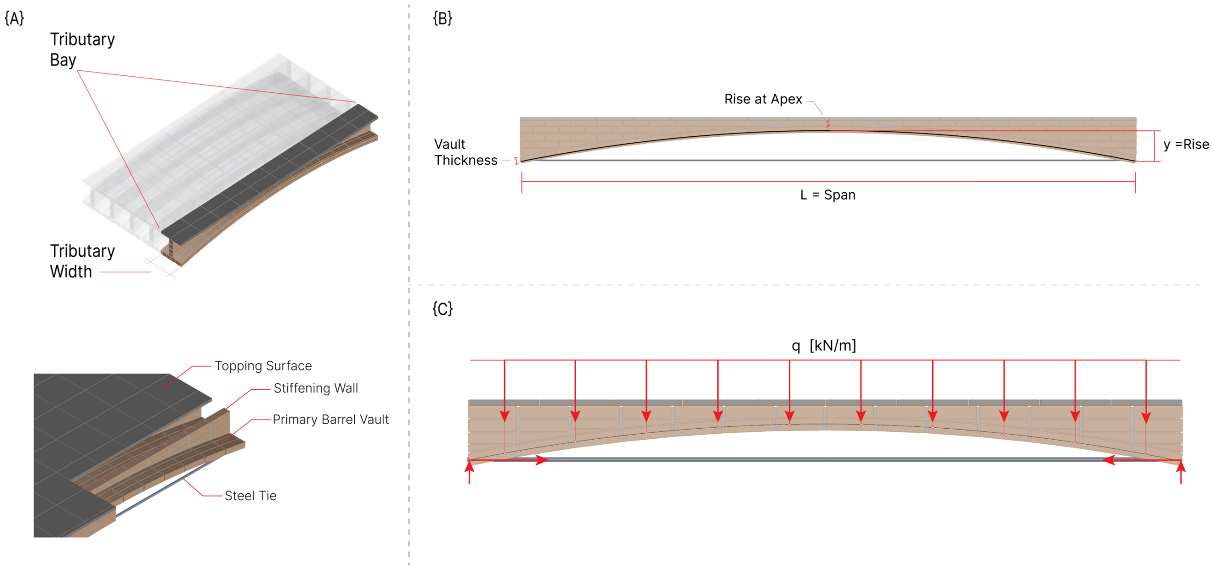

Figure 5: Earthen vaulted floor details and terminology

A 2D equilibrium analysis is performed on a single tributary bay for each floor system, as illustrated in Figure 6. This bay includes one stiffening wall, a segment of the primary barrel vault and topping surface equal to the tributary width. The tributary width corresponds to the spacing between stiffening walls: 1m for typology SR, 30cm for typology DR, and 1m for typology LF (chosen based on the absence of stiffening walls). As depicted in Figure 5, the vaults possess a uniform thickness of 5cm, with stiffening walls rising to a height of 5 cm at the center, referred to as the apex rise. Span measurement is taken from the centerline of the primary vault.

Loading conditions encompass self-weight, finishing loads, and factored live and dead loads in line with the International Building Codes (IBC) – which the Ethiopian Building Code is based on [17]. IBC specifies a $1.5\ kN/m^2$ dead load, $1.5\ kN/m^2$ partition load, and a $2\ kN/m^2$ live load; dead and partition loads subjected to a 1.35 safety factor and live loads 1.5. The loading in addition to self-weight applied to each typology is $SR = 8\ kN/m^2$, $DR = 7.5\ kN/m^2$, $LF = 7\ kN/m^2$. For analysis, all loading is linearly projected along the span of the tributary bay, assuming that loads are transferred vertically to the primary barrel vault. Given a parabolic shape of the primary barrel vault and linearly projected loads, the forces in the vault are computed as a three-hinge arch under uniform loading (q):

$$\text{Thrust} = \frac{qL^2}{8y} \tag{1}$$

$$\text{Vertical} = \frac{qL}{2} \tag{2}$$

Since the stiffening walls and fill are assumed to only transfer loads vertically, they are excluded from the tributary area. Thus, for calculating working stresses, the tributary area is taken as the tributary width multiplied by the thickness of the primary barrel vault.

A parametric model was created in Rhinoceros 3D to represent each floor typology. This model served as the basis for the described 2D equilibrium analysis. Each floor system was sampled in 0.01 meters interval over a 0.5 -10.5 meters range. Additionally, the analysis included two shallowness ratios, defined as the ratio of span to rise, with the first set to *span/10* and the second to *span/20*.

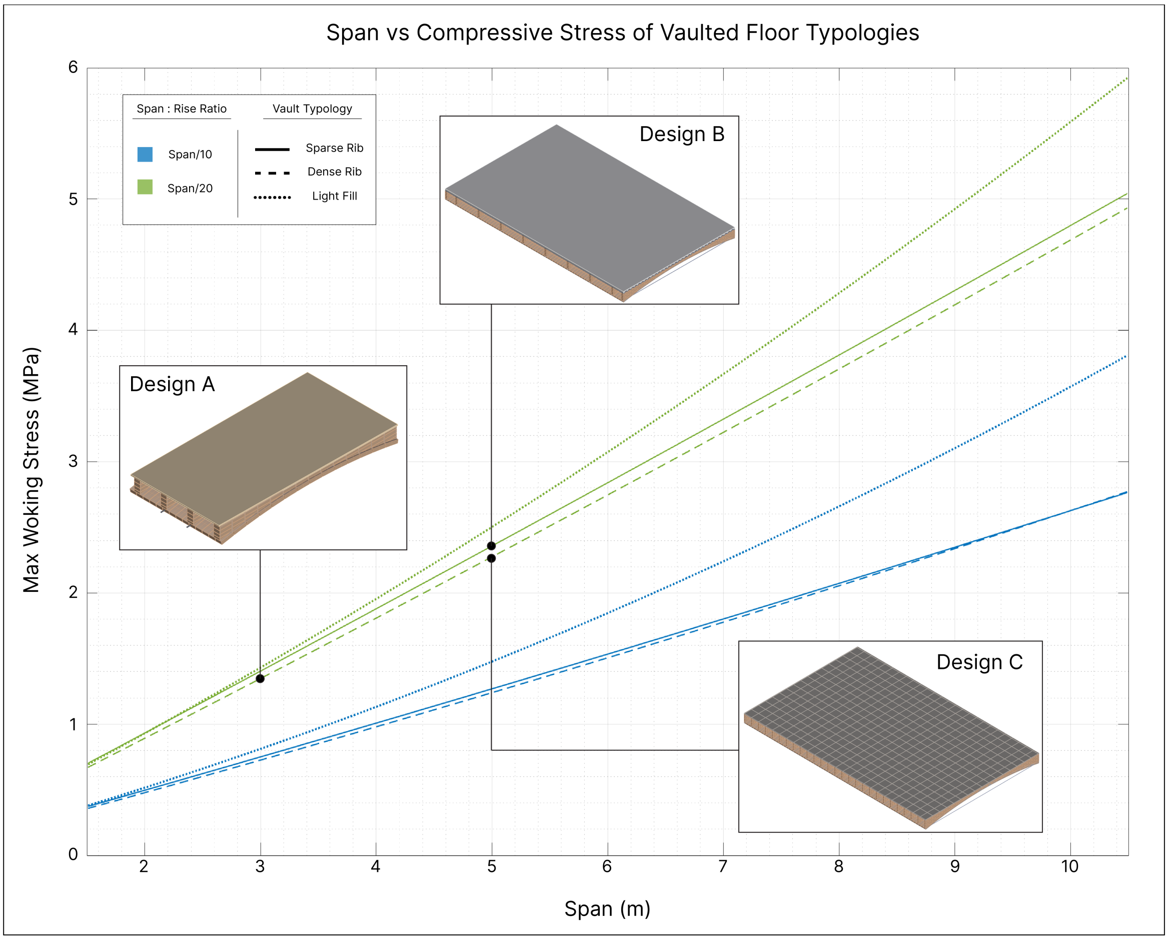

Figure 6: Working stresses of earthen vaulted floor typologies; following on the work of [18]

3.3 Results

Figure 6 illustrates that all floor typologies can span distances commonly found in mid-rise buildings while maintaining low maximum working stresses. For instance, designs B and C (Figure 6), show that even at a shallow aspect ratio of *span/20,* both designs span a 5-meter distance while keeping maximum stresses under 2.5 MPa, as per the assumptions in section 3.2. This underscores the feasibility of utilizing CSEBs as affordable, low-carbon masonry. The validity of these findings is further shown by the recently built 3-meter scale prototype to be discussed in Section 5, highlighted as Design A in Figure 6.

A notable aspect of the results depicted in Figure 7 is that low stresses are observed across various typologies, spans, and aspect ratios. While soil properties and CSEB strengths vary across different locations [13], the results indicate the feasibility of adapting the design to diverse contexts by adjusting the typology or aspect ratio. Although only *span/10* and *span/20* are plotted, additional ratios can be interpolated. Thus, as brick properties vary, a different typology can be selected, the span-to-rise ratio can be reduced, or vault thickness can be increased. Moreover, while this paper focuses on CSEBs, other readily available construction alternatives, such as masonry units made from construction waste or more traditional fired clay bricks can be employed in the proposed system.

While confirming the structural feasibility of earthen vaulted floor systems, these findings have limitations. The analysis is confined to static behavior, necessitating further exploration of seismic and unconventional load scenarios. Despite low stresses in Figure 7, CSEBs exhibit inherent variability, demanding heightened safety considerations. Design recommendations include a safety factor of 3 on CSEB strength. For maximum vault stress of 2.5 MPa, a CSEB with 7.5 MPa strength is advised. Additionally, a span-to-rise ratio of 1/20 allows little room for construction error, prompting a recommendation of 1/10 if concerns about construction quality arise.

4. Case Study

This research analyzes the cost and embodied carbon of two vaulted earthen floor designs in contrast to a flat slab within a recently completed 15-story building in Addis Ababa, Ethiopia. The floor designs are tailored for a 5x8 meter column grid spacing (40 m^2), reflecting Ethiopia's standard slab span in mid-rise affordable housing projects. Ethiopia is relevant to this investigation, given its high construction material costs, lower labor expenses, and substantial demand for affordable housing.

4.1 Slab & Floor Typologies

The first earthen slab, showcased in the top half of Figure 7, is a dense rib type with a 5-meter span and a span/20 shallowness ratio. The total thickness, including the 2.5 cm thick ceramic tile finishing surface, is 37 cm. The second earthen slab, depicted in the bottom half of Figure 7, is a sparse rib type with a span/20 shallowness ratio. Its total thickness, including the finishing surface, is 42 cm. The finish, visible on the bottom right of Figure 7, consists of a 1 mm thick corrugated steel deck and a 1400 kg/m^3 C25 3.5 cm concrete slab, resulting in a total thickness of 7 cm. The comparative slab, presented in Figure 8, is a C35 concrete, 17 cm thick two-way flat slab with a reinforcement ratio of approximately 0.7%.

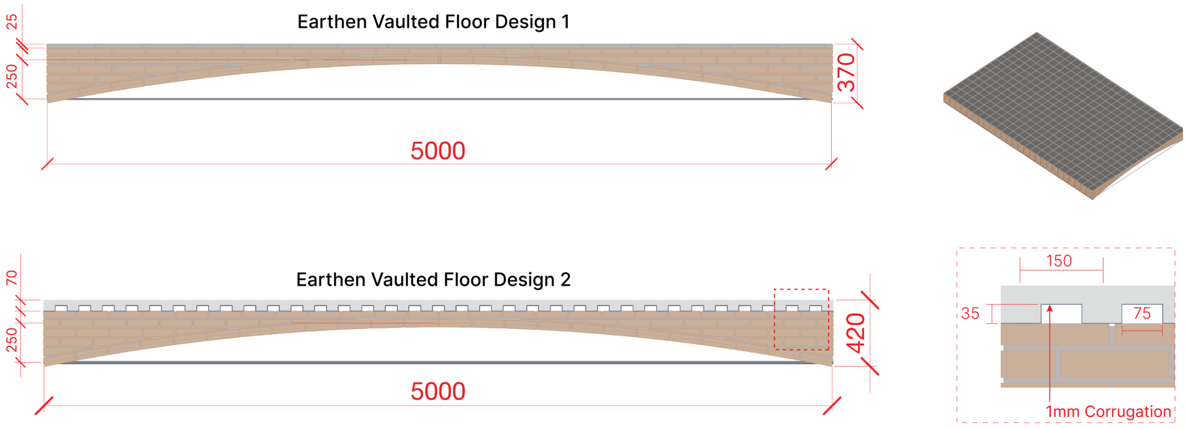

Figure 7: Earthen vaulted floor designs for case study (mm)

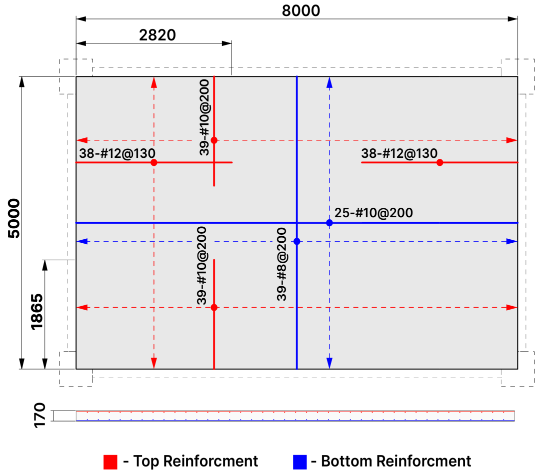

Figure 8: As built two-way flat slab details (mm)

4.2 Case Study Assumptions

Table 1 represents A1-A3 embodied carbon coefficients (ECC) of building materials in $kg_{CO2e}/unit$. To obtain values that most closely relate to Ethiopia, the Rwanda Embodied Carbon Calculator (RwECC) is used [19]. The Inventory of Carbon & Energy database (ICE) is used for unavailable values [20]. With respect to CSEB choice, the author produced 3% and 7% cement stabilized bricks with a strength of 5+ MPa in Ethiopia thus the value in Table 1 is deemed a fair estimate of ECC for a CSEB necessary for the proposed designs [21].

The material and labor costs presented in Table 1 were obtained through interviews with local engineers and data collected from CSEBs testing in Ethiopia. Formwork costs are conservatively taken to be the same cost for both vaults and concrete slabs. Regarding masonry labor, the local rate for masonry work is \$2/m^2. However, due to the increased complexity of vault construction, this is assumed to increase to \$6/m^2. Lastly due to variation, concrete labor costs are estimated to constitutes 35% of the total structural which was derived from an interview the author had with engineer Yonas Derbie's audit of 15 recently constructed buildings.

Table 1: Embodied Carbon Coefficients and Costs of Materials and Labor

Material / Item	Density [kg/m^3]	ECC [kg_{CO2e}]	Source	Unit for Cost	\$ USD
C35 Concrete	2400	339 / m^3	RwECC	m^3	\$241
LW C25 Concrete	1400	270 / m^3	RwECC	m^3	\$185
1:3 Mortar	1800	0.2 / kg	ICE	m^3	\$139
Steel Corrugation	7850	1.44 / kg	RwECC	m^2	\$31
Steel Tie (Rebar)	7850	1.2 / kg	RwECC	kg	\$2
5% CSEB	2000	0.04 / kg	RwECC	m^3	\$93
Clay Tile	2000	0.22 / kg	RwECC	m^2	\$8
Formwork*	-	-	-	m^2	\$30
Masonry Labor*	-	-	-	m^2	\$6
Concrete Labor*	-	-	-	%	35%

4.3 Case Study Results

Figure 9 shows that both design cases have significant reductions in cost, embodied carbon, and weight. ***Design 1 shows a 57% reduction in weight, a 74% reduction in embodied carbon, and a 62% reduction in cost; likewise, Design 2 respectively shows a 52%, 50%, and 42% reduction***. While already significant, the notable reduction in weight achieved by both designs suggests that the size and mass of columns and foundations can be reduced, providing even further savings in embodied carbon and cost.

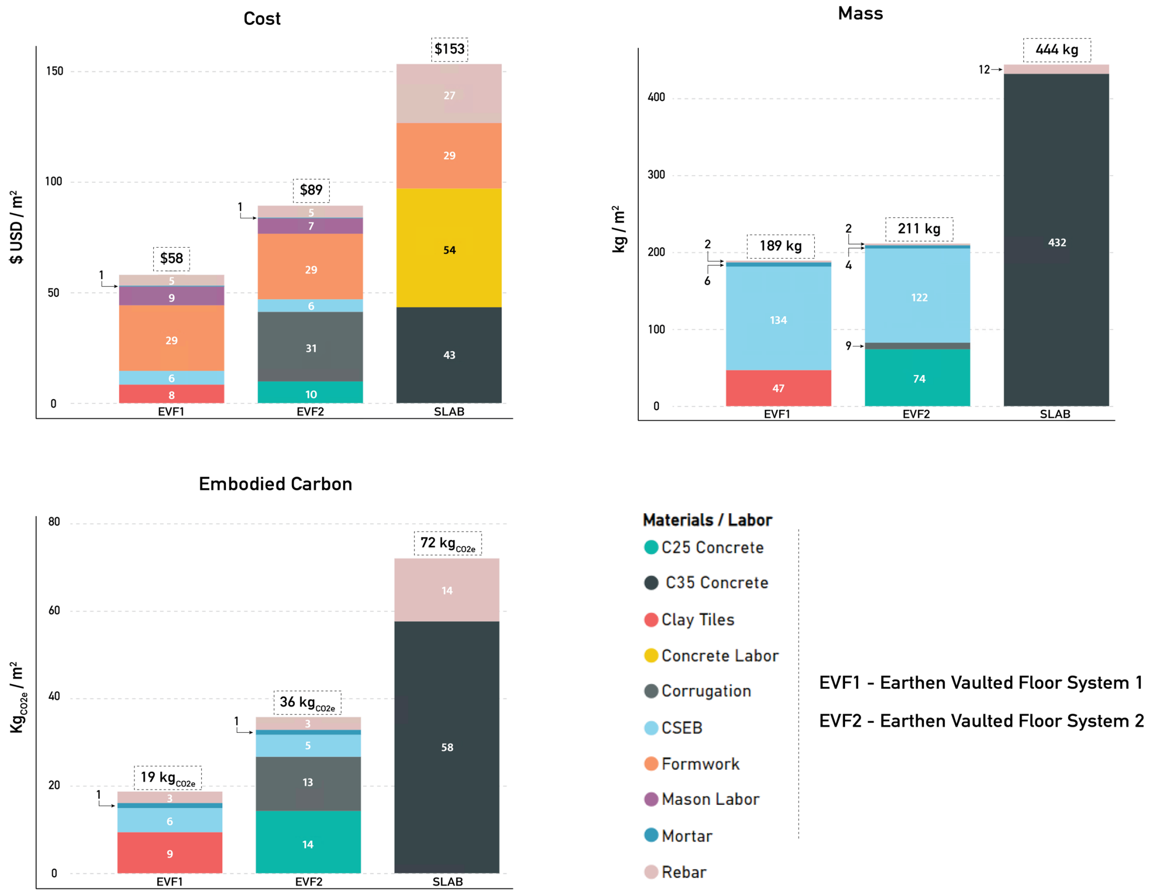

Figure 9: Cost, Carbon, Mass Breakdown of Case Study Floor Systems

5. Physical Prototype

5.1 General Design

The prototype (Figure 10) is 3 meters in span, 1.5 meters in depth, and 5 cm thick, with a span-to-rise ratio of 1/20 giving a primary vault depth of 15 cm. It has stiffening walls spaced at 50 cm and an apex rise of 5 cm. The primary vault and the stiffening walls were constructed with a 3% cement-stabilized Compressed Stabilized Earth Block (CSEB) of 11 MPa strength. The finishing surface was a 20 mm thick plywood piece spanning stiffening walls without attachment. The entire vault rested on steel angles, connected by steel ties at each stiffening wall to manage thrust.

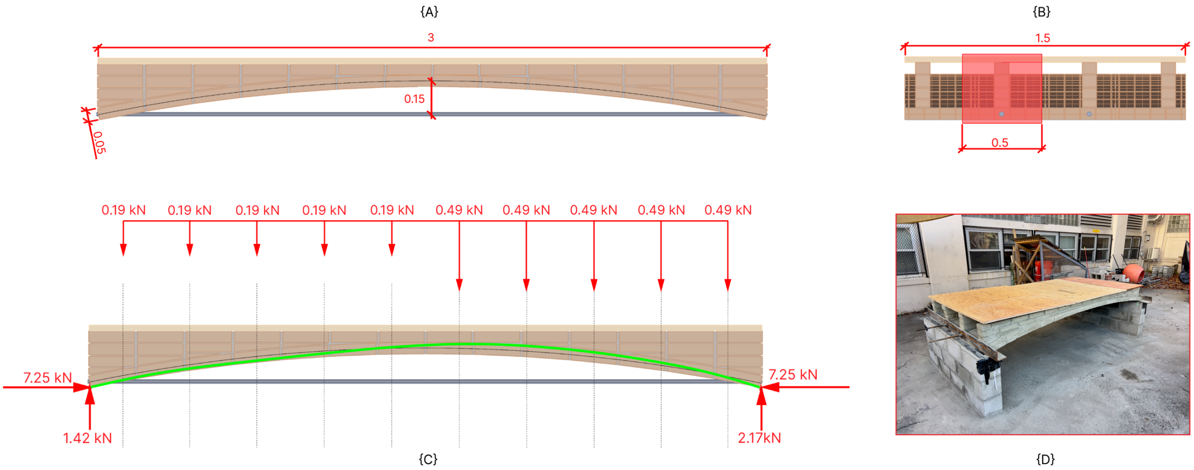

Figure 10: Building dimensions (m) and loading details of 3m vault prototype.

5.2 Prototype Structural Analysis

Following the procedures in Section 4.2, a 2D equilibrium analysis was performed. The prototype was assessed for an unfactored live load of 2 kN/m^2 despite its higher capacity, as evident in Figure 6. Testing was conducted symmetrically and asymmetrically to demonstrate safety under more complex loads, with the latter considered the critical case for stability. Under symmetrical loading, stresses were 0.5 MPa, well below CSBE capacity. The thrust line fitting within the primary vault (Figure 10-C) during asymmetrical loading confirmed stability per Heyman's Safe theorem [8].

The 2D analysis was corroborated through a 3D equilibrium thrust network analysis. A Rhino 3D model of the prototype's intrados and extrados surfaces was created, and a 3D equilibrium analysis was carried out using The Thrust Network Optimization (TNO) plugin by BRG [22]. TNO confirmed the 2D analysis, while also showing a thrust reduction from 7.3 kN to 6.4 kN under asymmetrical loading while maintaining stability (Figure 11). Both analyses conservatively assume thrust lines can't enter stiffening walls. However, proper construction would permit this, giving a deeper thrust line and lower stresses.

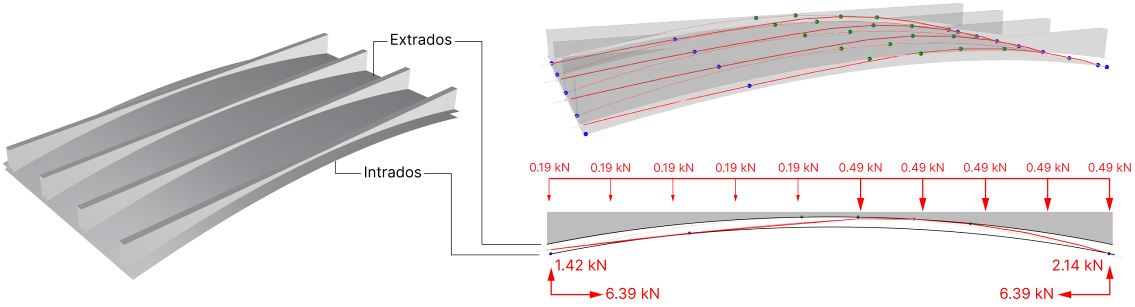

Figure 11: 3D TNO Analysis of 3m Prototype

5.3 Prototype Testing

For testing, the prototype was loaded in a symmetrical sequence with fifty 20-liter buckets of water, totaling 9.27 kN over the 4.5 m^2 area, achieving the desired 2 kN/m^2. Asymmetrical loading cases involved removing buckets from one-half of the vault. The prototype withstood symmetrical and asymmetrical loads for a sustained period, including two human point loads (1.25 kN total) at the vault center (Figure 12). A comprehensive analysis of the prototype is provided in the first author's master's thesis [21].

Figure 12: (a) Prototype under uniform load of 2 kN/m^2; (b) uniform load and human point load totaling 1.25 kN

6. Closing Remarks

This paper introduces three earthen vaulted floor typologies that utilize shell behavior to efficiently span typical building spans while keeping stresses below 5 MPa, aligning with typical CSEB strengths. These systems demonstrate cost and embodied carbon reductions of up to 62% and 74%, respectively. The study shows the initial feasibility of the earthen shell floor systems through prototyping and testing.

Future research is needed on the seismic safety of these systems verified with more in depth physical prototyping. Furthermore, as the overarching objective is the integration of these systems into existing mid-rise concrete frame buildings, additional efforts are required to understand the construction process and structural behavior at the building scale. Similarly, broadening the case study to encompass downstream effects on columns, beams, and foundations will provide a more comprehensive understanding of cost implications and carbon savings at the whole building scale.

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